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AN OPTIMIZATION STUDY OF EFFECTS ON AIRCRAFT PERFORMANCE OF VARIOUS FORMS OF HEAT ADDITION

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SUMMARY

Basic ram-jet aircraft design considerations are reviewed at a level of simplification appropriate for evaluation of external heat addition schemes. No definite conclusions are given as to the relative advantage of external combustion in comparison with conventional ram-jet combustion because of the incompleteness of the knowledge of both at hypersonic speeds. Instead, similarity parameters are derived which will allow a ready comparison when complete data become available. Possible variations of quantities, such as wing size relative to engine size, which would affect the comparison are eliminated from consideration by deriving the optimum values.

INTRODUCTION

Conventionally it has been possible in performance analyses to consider wing-fuselage combinations and engines relatively independently. When external combustion or internal lift is considered, as would be expected, the interaction between components increases. In such cases maximum performance will not be achieved by merely maximizing the wing lift to drag ratio, nor the engine over-all efficiency. Further the quantities which should be maximized depend to a greater extent than formerly on the particular aircraft mission under consideration.

In the present report the simplest meaningful mathematical model which could be found for ram-jet configurations has been developed for the purpose of evaluating various heat addition schemes for high flight speeds. It is not possible to complete such evaluations at this time because of the absence of necessary data. However, it is believed that a contribution toward analyzing the problem has been made which will be useful when the necessary data become available.

To define performance quantitatively, it is necessary to specify whether range capability or acceleration capability is to be maximized. After this step, expressions for the performance in terms of basic design parameters are adopted. The expressions selected are justified by reasoning based on the use of reversible heat addition theory and the use of dimensionless ratios.

In order to simplify the comparisons, several of the design variables, such as the ratio of wing size to engine size, are eliminated from consideration since the optimum values are derived. After discussing the properties of conventional ram-jet configurations with the aid of the simplified model, it is shown that the effect on performance of underwing heat addition can be expressed in terms of an effective lift to drag ratio. Expressions for this quantity are derived in the two cases considered (range and acceleration). The present model can be applied to other types of heat addition; the performance possibilities of rocket ram-jet configurations are discussed briefly.

The results of previous investigations of the effects on airplane performance of combustion under wings are reported in references 1 to 8. These analyses are based on a theory of heat addition developed in references 9 to 12.

In references 1 and 2, a graphical method for solving the basic equations in the two-dimensional case is developed. This theoretical study was followed up by experiments conducted at the NASA Lewis Research Center. The results of these experiments are reported in references 13 to 17.

SIMPLIFIED PERFORMANCE THEORY

In this section, the problem of analyzing the performance of aircraft which employ air-breathing propulsion for accelerating to high velocity or for steady powered flight is reviewed.

For present purposes, the flight path is divided into three parts which are treated as independent missions. These are: (1) acceleration to top speed, which from rocket terminology is called the burnout velocity mission; (2) the part of the range achieved during steady powered flight as described by the original Breguet range equation; and (3) the unpowered glide.

Since only speeds below half of satellite speed will be considered, the orbital centrifugal force will be neglected. The aerodynamic forces will be resolved into horizontal and vertical components and only horizontal motion will be considered. Methods for correcting these approximations will be noted but will not be included in the development.

The forces due to the fuel mass flow will be neglected; that is, the effect of fuel injection will be taken to be simple heat addition as in references 9 to 12. For comparison, relations applicable to rockets will be cited wherein the forces due to fuel mass flow are not neglected since they are dominant.

Burnout Velocity Mission

In horizontal flight, the thrust and acceleration of a rocket or ram-jet configuration are related by the expression

$$T = \frac{W}{g}\frac{dV}{dt} \tag{1}$$

where T is the net horizontal thrust as distinguished from the usual engine thrust, W is the weight of the configuration and V is the velocity.

For a particular accelerated flight, the mass of the configuration W/g is a function of the velocity, and we wish to determine this function. Equation (1) can be written as

$$T = \frac{1}{g}\frac{dV}{d(ln\ W)}\frac{dW}{dt} \tag{2}$$

The quantity $T/-(dW/dt)$ is equal to the effective specific impulse, I, so that equation (2) becomes

$$\frac{dV}{d(ln\ W)} = -Ig$$

For constant specific impulse, this can be integrated to obtain the well-known rocket relation

$$V - V_i = Ig\ ln\frac{W_i}{W} \tag{3}$$

where V_i and W_i are initial values (see, e.g., ref. 18). For a ram-jet configuration, the specific impulse eventually decreases as the velocity increases. In the range of efficient operation of a ram jet, the quantity which is more nearly constant is the over-all airplane efficiency η which is defined by the relation

$$\eta = \left(\frac{TV}{Q}\right)_{L=W} \tag{4}$$

where T is the net thrust (engine thrust minus airplane drag), Q is the total heat energy added to the air per unit time (ft-lb/sec), and L is the total lift.

The present notation for heat addition, taken from references 12 and 19, differs from the usual engine terminology. However, it has several advantages after familiarity with it is achieved. One advantage is the use of dimensionless ratios, common in aerodynamics. Another is the use of the theory of reversible heat addition which leads to conclusions which are otherwise not so apparent. For example, if the forces due to fuel mass flow are neglected, the first and second laws of thermodynamics require that the over-all airplane efficiency be less than 1. No violation of this condition is to be expected for air-breathing configurations at speeds below half of satellite velocity. On the other hand, if the over-all airplane efficiency is much less than 1/3, it is probable that some rearrangement of components can be found which will increase the efficiency. Consequently, there is reason to conclude that this efficiency should be roughly constant in the range of efficient operation of air-breathing configurations.

In terms of the over-all airplane efficiency, equation (2) can be written as

$$\eta = \frac{V[dV/d(\ln W)][d(W/g)/dt]}{Q} \tag{5}$$

The quantity $-Q/[(dW/g)/dt]$ can be recognized as the heat content per unit mass of the fuel (see ref. 19) and can be expressed as

$$-\frac{Q}{(dW/g)/dt} = \eta_c k \frac{1}{2} V_{sat}^2 \tag{6}$$

where η_c is the combustion efficiency (a number between 0 and 1), k is a dimensionless fuel heat content parameter, and V_{sat} is satellite velocity (26,100 ft/sec). The values of k for gasoline and hydrogen (ref. 20) are

$$k = \begin{cases} 1.4 \text{ (gasoline)} \\ 3.9 \text{ (hydrogen)} \end{cases} \tag{7}$$

In the present terminology the total heat addition rate (Q) is used in the place of the fuel mass flow $-d(W/g)/dt$. Equations (6) and (7) can be used to convert to the usual engine terminology.

Equations (5) and (6) can be combined to obtain the relation

$$V \frac{dV}{d(\ln W)} = -k \frac{1}{2} V_{sat}^2 \eta_c \eta \tag{8}$$

For a constant value of the product $\eta_c \eta$, equation (8) can be integrated to determine the desired relation between velocity and mass ratio which is the expression

$$\left(\frac{V}{V_{sat}}\right)^2 - \left(\frac{V_i}{V_{sat}}\right)^2 = k\eta_c \left(\frac{TV}{Q}\right)_{L=W} \ln \frac{W_i}{W} \tag{9}$$

From this result, it follows that for a given fuel heat content parameter, k, given combustion efficiency, η_c, and given mass ratio, the burnout velocity is maximized if the over-all airplane efficiency $\eta = (TV/Q)_{L=W}$ is a maximum at all stages of the acceleration.

In practice the over-all airplane efficiency would not be constant over a large velocity increment and equation (9) would not be the correct integral of equation (8). However, it is convenient to divide the accelerated flight into parts corresponding to several velocity increments and to give consideration to the requirements for maximizing the over-all airplane efficiency in each velocity increment.

For flight at constant altitude and small velocity increments, the orbital centrifugal force can be taken into account by replacing W in the over-all airplane efficiency by the actual vertical force which is equal to the quantity

$$\left[1 - \left(\frac{V}{V_{sat}}\right)^2\right] W$$

Slowly varying altitude changes can be taken into account as energy equivalent velocity changes by subtracting from the left side of equation (9) the quantity

$$\frac{2hg}{V_{sat}^2} - \frac{2h_i g}{V_{sat}^2}$$

where h is the altitude.

Range Mission

The Breguet range equation (see, e.g., ref. 19) can be written as

$$\text{Range} = k\eta_c\left(\frac{LV}{Q}\right)_{T=0} ln \frac{W_i}{W} \times 2000 \text{ miles} \tag{10}$$

where k is the fuel heat content parameter evaluated in equation (7) and η_c is the combustion efficiency. The product $\eta_c(LV/Q)_{T=0}$ is an efficiency factor which is usually written as the product $\eta_e(L/D)$, where η_e is the over-all engine efficiency. However, when external heat addition or engine exhaust deflection is considered, it is not possible to factor $\eta_c(LV/Q)_{T=0}$ in this way. Equation (10) does not include the mass ratio required to accelerate to the cruise velocity nor the range achieved during the unpowered glide, since these are considered as separate missions.

If a rocket motor were used in the place of an air-breathing engine for sustained flight at constant altitude and velocity, equation (10) would become

$$\text{Range} = 2 \frac{IgV}{V_{sat}^2} \frac{L}{D} ln \frac{W_i}{W} \times 2000 \text{ miles} \tag{11}$$

Again equation (11) does not include the mass ratio required to accelerate to the cruise velocity, nor the range achieved during the unpowered glide. However, the factor $2(IgV/V_{sat}^2)(L/D)$ for the winged rocket does correspond to the factor $k\eta_c(LV/Q)_{T=0}$ for the air-breathing configuration, and hence is of interest for comparison with heat addition schemes.

The factor $k\eta_c(LV/Q)_{T=0}$ for ram jets is expected to decrease with increasing vehicle speed or perhaps, with development, a constant value of about 2.8 can be achieved at high velocities with gasoline as the fuel. The factor $2(IgV/V_{sat}^2)(L/D)$ for the rocket should increase almost linearly with velocity and reach a value of 2.8 at about half of satellite speed.

If the orbital centrifugal force is taken into account, the powered range is increased by the factor $1/[1 - (V/V_{sat})^2]$ in equations (10) and (11).

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ANALYSIS OF EFFICIENCIES

In general, we wish to consider an arbitrary hypersonic air-breathing configuration with arbitrary heat addition in order to determine criteria for maximizing the range in one case and for maximizing the burnout velocity in a second case.

Strictly, the mass ratio should be factored into a product of the ratio of initial to final weight times the ratio of final weight to payload weight (ref. 21). The burnout velocity or range should be maximized at a given ratio of initial to payload weight. For rockets the ratio of final weight to initial volume is considered more important than the mass ratio in some cases (ref. 22). However, for present purposes we will use the simple mass ratio as a measure of performance.

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If different methods of heat addition were used at different regions of the flow, the differing values of combustion efficiency would be involved in the optimization procedure. Similarly, if several fuels were used, the values of k would be involved. For simplicity, we will at first assume the same heat content and combustion efficiency for all fuel so that the optimum burnout velocity and range correspond to maximums of $(TV/Q)_{L=W}$ and $(LV/Q)_{T=0}$.

As an example of the procedure, and also to provide an approximate model for comparison, a configuration consisting of a ram-jet engine and a wing will be analyzed from a simplified point of view. It will be assumed that all the air which passes through the engine undergoes the same thermodynamic cycle and exhaust deflection. The losses caused by spillage and bluntness at the inlet and boundary layer bleed and friction drag are treated as engine drag, while internal shock losses are assumed to be uniform and are included in the thermodynamic cycle by means of a kinetic energy efficiency parameter (ref. 23).

The total forces and heat power can be approximated by the following expressions:

$$L = \rho VA\left[V_e\left(1 + \frac{p_e}{\rho_e V_e^2}\right)\right]\sin\theta + L \tag{13}$$

$$T = \rho VA\left\{\left[V_e\left(1 + \frac{p_e}{\rho_e V_e^2}\right)\right]\cos\theta - \left[V\left(1 + \frac{p}{\rho V^2}\right)\right]\right\} - (A_e - A)p - D_{eng} - D \tag{14}$$

$$Q = \rho VA\left(h_e + \frac{1}{2}V_e^2 - h - \frac{1}{2}V^2\right) \tag{15}$$

All symbols are defined in appendix A. The quantity ρVA is the air mass flow through the engine, fuel mass flow being neglected. Equations (13) and (14) are based on the assumption that the engine exit plane is perpendicular to the exit velocity vector, and that the engine inlet is at zero angle of attack.

An expression is needed for the energy added per unit mass in the internal flow (the quantity in parentheses in eq. (15)) in terms of the quantities $V_e[1 + (p_e/\rho_e V_e^2)]$ and $V[1 + (p/\rho V^2)]$ which appear in the expressions for the forces. We assume the following relation:

$$\frac{1}{2}\left[V_e\left(1 + \frac{p_e}{\rho_e V_e^2}\right)\right]^2 = \eta_t\left(h_e + \frac{1}{2} V_e^2 - h - \frac{1}{2} V^2\right) + \eta_k \frac{1}{2}\left[V\left(1 + \frac{p}{\rho V^2}\right)\right]^2 \tag{16}$$

The reasons for this assumption are as follows: Equations (13) and (14) indicate that the quantity $V_e[1 + (p_e/\rho_e V_e^2)]$ varies linearly with the lift and thrust for a fixed value of θ. Hence it is desirable to utilize as large a fraction as possible of the fuel energy toward increasing this quantity. The quantity η_t is a measure of the efficiency of this conversion. Also it is desirable to conserve as much as possible of the free-stream value of $V[1 + (p/\rho V^2)]$. The quantity η_k is a measure of the efficiency of this conservation. Equation (16) is the simplest possible relation which has the foregoing properties. The squares are necessary in order that η_t and η_k be dimensionless, and in order that equation (16) will be an energy equation in the limit of large inlet and exit Mach numbers. In this approximate treatment of ram-jet and turbojet engines, it is not feasible to use a more complicated relation. For a particular engine, the quantities η_t and η_k may vary with thrust coefficient, but the values should lie between 1/2 and 1.

Insertion of equation (16) into (15) yields the relation

$$Q = \rho VA \frac{1}{\eta_t}\left\{\frac{1}{2}\left[V_e\left(1 + \frac{p_e}{\rho_e V_e^2}\right)\right]^2 - \eta_k \frac{1}{2}\left[V\left(1 + \frac{p}{\rho V^2}\right)\right]^2\right\} \tag{17}$$

By rearrangement, equations (13), (14), and (17) can be written as follows:

$$L = \frac{1}{2} \rho V^2 A\left(2 \frac{\tilde{V}_e}{V} \sin \theta + \frac{L}{D} C_D \frac{S}{A}\right) \tag{18}$$

$$T = \frac{1}{2} \rho V^2 A\left\{2\left[\frac{\tilde{V}_e}{V} \cos \theta - \left(1 + \frac{1}{2} \tilde{C}_{D_{eng}}\right)\right] - C_D \frac{S}{A}\right\} \tag{19}$$

$$Q = \frac{1}{2}\,\rho V^2 AV\left\{\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right]\right\} \tag{20}$$

where

$$\frac{\tilde{V}_e}{V} = \frac{V_e[1 + (p_e/\rho_e V_e{}^2)]}{V} \tag{21}$$

$$\tilde{C}_{D_{eng}} = C_{D_{eng}} + \frac{A_e}{A}\,\frac{p}{(1/2)\rho V^2} \tag{22}$$

$$\tilde{\eta}_k = \eta_k\left(1 + \frac{p}{\rho V^2}\right)^2 \tag{23}$$

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If the quantities $\tilde{V}_e/V$, θ, and S/A are assumed to be independent variables, and the parameters L/D, η_t, $\tilde{\eta}_k$, C_D, $\tilde{C}_{D_{eng}}$ are taken to be known constants, equations (18) to (20) form a self-consistent system from which considerable insight into the applications and limitations of ram-jet and turbojet airplanes can be obtained.

Although several of the quantities appearing in equations (18) to (20) are not free of ambiguity as defined, and these relations have been derived only by approximation, it may be possible to establish them on a semi-empirical basis. For example the exact definition of wing plan-form area (S) should be such that the total lift varies linearly with S, and that the coefficient of this variation, $(1/2)\rho V^2(L/D)C_D$, is independent of $\tilde{V}_e/V$, θ, and S/A. This may not always be possible. If not, additional terms would be needed in equations (18) to (20). In some cases, it may be possible to maintain the simplicity of these equations by varying geometric parameters which do not appear explicitly, such as wing camber or plan form. From the present point of view, the motive for such variations would be to establish a simple model which can be rigorously analyzed and which contains most of the essentials of the problem, rather than for any demonstrable improvement in performance.

Similarly the engine drag should include that portion of the external drag which is independent of the wing plan-form area.

The variation of exhaust velocity with heat power can always be approximated by a relation of the form of equation (20) in the neighborhood of a given design point. The constants η_t and $\tilde{\eta}_k$ can then be determined using equation (20) or equation (16) as a definition. Further details of this procedure are given in appendix B.

Once the expressions (18), (19), and (20) for the total forces and power are established, it is a straightforward procedure to form the particular efficiency ratio under consideration, $(LV/Q)_{T=0}$ for range or $(TV/Q)_{L=W}$ for burnout velocity. The resulting expressions can then be maximized with respect to the design variables $\tilde{V}_e/V$, θ, and S/A for fixed values of the design constants.

In practice, the design variables may not be optimized for reasons not considered here, but for present purposes optimization serves to remove these variables from consideration and hence leads to simplification in the evaluation of heat addition schemes.

Range Mission

The efficiency ratio for the range mission can be written, by means of equations (18), (19), and (20), as the relation

$$\left(\frac{LV}{Q}\right)_{T=0} = \left\{ \frac{2(\tilde{V}_e/V)\sin\theta + (L/D)C_D(S/A)}{(1/\eta_t)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]} \right\}_{T=0} \tag{24}$$

The condition $T = 0$ can be used to eliminate S/A from equation (24), that is, it is seen from equation (19) that $T = 0$ if the relation

$$\frac{S}{A} = \frac{2\left\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\right\}}{C_D} \tag{25}$$

is satisfied.

Subsequent developments are based on the assumption that the ratio of wing size to engine size will be varied to agree with equation (25) when $\tilde{V}_e/V$ and θ are varied. The results to be given will apply if this condition is relaxed slightly. Quantitative estimates of the effect of such departures can be made using the basic mathematical model represented by equations (18), (19), and (20). However, this point will not be further considered in the present report.

Substitution of equation (25) into (24) yields

$$\left(\frac{LV}{Q}\right)_{T=0} = 2\eta_t \left(\frac{(\tilde{V}_e/V)\sin\theta + (L/D)\left\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\right\}}{(\tilde{V}_e/V)^2 - \tilde{\eta}_k}\right) \tag{26}$$

By differentiation, it is found that $(LV/Q)_{T=0}$ is a maximum for a fixed value of $\tilde{V}_e/V$ if θ satisfies the relation

$$\tan \theta_{opt} = \frac{1}{L/D} \tag{27}$$

and substitution of this value into equation (28) yields

$$\left(\frac{LV}{Q}\right)_{T=0} = 2\eta_t \left\{ \frac{\sqrt{1 + (L/D)^2}(\tilde{V}_e/V) - (L/D)[1 + (1/2)\tilde{C}_{D_{eng}}]}{(\tilde{V}_e/V)^2 - \tilde{\eta}_k} \right\} \tag{28}$$

Finally optimization with respect to $\tilde{V}_e/V$ leads to the relations

$$\left(\frac{\tilde{V}_e}{V}\right)_{opt} = \sqrt{\tilde{\eta}_k}\left(\frac{[1 + (1/2)\tilde{C}_{D_{eng}}](L/D)}{\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}} + \sqrt{\left\{\frac{[1 + (1/2)C_{D_{eng}}](L/D)}{\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}}\right\}^2 - 1}\right) \tag{29}$$

$$\left(\frac{LV}{Q}\right)_{T=0_{opt}} = \frac{\eta_t\sqrt{1+(L/D)^2}/\sqrt{\tilde{\eta}_k}}{\left\{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)/\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}\right\}+\sqrt{\left\{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)/\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}\right\}^2-1}} \tag{30}$$

From equation (30) it is seen that the optimum range efficiency depends only on the two similarity parameters $\eta_t\sqrt{1 + (L/D)^2}/\sqrt{\tilde{\eta}_k}$ and $[1 + (1/2)\tilde{C}_{D_{eng}}](L/D)/\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2} = x_R$. For present purposes $\tilde{\eta}_k$ can be approximated by the kinetic energy efficiency defined in reference 23. Values of diffuser kinetic energy efficiency from 0.9 to 1.0 are possible up to a Mach number of 5. Thermodynamic cycle efficiencies (η_t) of 0.5 are typical for ram jets. An L/D of 6 may be possible at a Mach number of 5. Equation (30) is plotted in figure 1. With the parameter $\eta_t\sqrt{1 + (L/D)^2}/\sqrt{\tilde{\eta}_k}$ equal to a typical value of 3, the values of $(LV/Q)_{T=0}$ would be three times the ordinate shown in figure 1. It can be seen in figure 1 that the range efficiency factor $(LV/Q)_{T=0}$ decreases monotonically as the loss parameter, x_R, is increased, for a fixed value of the parameter $\eta_t\sqrt{1 + (L/D)^2}/\sqrt{\tilde{\eta}_k}$.

Substitution of equations (27) and (29) into (25) yields

$$\left(\frac{S}{A}\right)_{opt} = \frac{2\sqrt{\tilde{\eta}_k}}{C_L}\frac{(L/D)^2}{\sqrt{1 + (L/D)^2}}\left[\sqrt{x_R^2 - 1} - \frac{x_R}{(L/D)^2}\right] \tag{31a}$$

for the optimum ratio of wing plan-form area to engine inlet capture area.

For all values of the loss parameter, except those very near 1.0, this can be approximated by the relation

$$\left(\frac{S}{A}\right)_{opt} = \frac{2\sqrt{\tilde{\eta}_k}}{C_L}\,\frac{L}{D}\sqrt{x_R^2 - 1} \tag{31b}$$

Equation (31b) is plotted in figure 2. With the parameter $\sqrt{\tilde{\eta}_k}(L/D)/C_L$ equal to a typical value of 40, the values of S/A would be 40 times the ordinate shown in figure 2.

If because of low engine losses or low L/D, the loss parameter is less than the quantity $(L/D)^2/\sqrt{(L/D)^4 - 1}$, then $(S/A)_{opt}$ is 0, and a different optimization procedure starting from equation (24) is required. This case will not be further considered here.

The heat power required to produce the optimum value of range efficiency can be obtained from equation (20). It is convenient to express the heat power as a coefficient defined by the relation

$$C_Q = \frac{Q}{(1/2)\rho V^2 AV} = \frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right] \tag{32}$$

With the aid of equation (6) it can be seen that the heat power coefficient is related to the fuel mass flow $d(-W/g)/dt$ by the relation

$$\frac{d(-W/g)}{dt} = \frac{\rho AV}{\eta_c k}\left(\frac{V}{V_{sat}}\right)^2 C_Q \tag{33}$$

Substitution of equation (29) into (32) yields

$$C_{Q_{opt}} = \frac{2\tilde{\eta}_k}{\eta_t}\sqrt{x_R^2 - 1}\left(x_R + \sqrt{x_R^2 - 1}\right) \tag{34}$$

for the optimum power coefficient. This relationship is plotted in figure 3.

For a given fuel, there is a maximum value of C_Q corresponding to the stoichiometric ratio, given approximately by the relations

$$C_{Q_{max}} = \alpha\eta_c\left(\frac{V_{sat}}{V}\right)^2 \tag{35}$$

$$\alpha \cong \begin{cases} 1/13 & \text{gasoline} \\ 1/9 & \text{hydrogen} \end{cases}$$

where V_{sat} is satellite velocity (26,100 ft/sec) and η_c is the combustion efficiency. Because of this limit on C_Q, it may not be possible to achieve the optimum value indicated by equation (34). In that case, the variation of the efficiency with C_Q is of interest. Substitution of equation (32) into (28) yields

$$\left(\frac{LV}{Q}\right)_{T=0} = \frac{2\eta_t\sqrt{1 + (L/D)^2}}{\sqrt{\tilde{\eta}_k}}\left[\frac{\sqrt{1 + (\eta_t/\eta_k)C_Q} - x_R}{(\eta_t/\eta_k)C_Q}\right] \tag{36}$$

Figure 4 is a plot of the efficiency for several values of $(\eta_t/\tilde{\eta}_k)C_Q$ including the optimum value previously given in figure 1. The corresponding values of S/A are obtained by substitution of equations (27) and (31a) into (26) which results in the expression

$$\frac{S}{A} = \frac{2\sqrt{\tilde{\eta}_k}}{C_L}\frac{(L/D)^2}{\sqrt{1 + (L/D)^2}}\left[\sqrt{1 + \frac{\eta_t}{\tilde{\eta}_k}C_Q} - \frac{1 + (L/D)^2}{(L/D)^2}x_R\right] \tag{37a}$$

This can be approximated by the relation

$$\frac{S}{A} = \frac{2\sqrt{\tilde{\eta}_k}\,(L/D)}{C_L}\left(\sqrt{1 + \frac{\eta_t}{\tilde{\eta}_k}C_Q} - x_R\right) \tag{37b}$$

Figure 5 is a plot of S/A for several values of $(\eta_t/\tilde{\eta}_k)C_Q$.

It can be seen in figure 5 that although the optimum value of the ratio of wing size to engine size S/A increases with increasing values of the loss parameter, if there is a maximum power coefficient less than the optimum power coefficient, this trend is reversed.

Since the factor $(L/D)/\sqrt{1 + (L/D)^2}$ usually differs from 1 by only a few percent, in many cases the engine parameters can be optimized independently of the wing parameters with little error. In such cases, the engine exhaust deflection can be set equal to 0 with a loss of only a few percent in over-all range efficiency. However, if the quantity $[1 + (1/2)\tilde{C}_{D_{eng}}]/\sqrt{\tilde{\eta}_k}$ also differs from 1 by only a few percent, or the optimum power coefficient cannot be reached, the steep slopes in figure 4 indicate that the engine exhaust deflection can be important.

The results of equation (36) and figure 4 summarize in convenient terms several results which could be obtained only laboriously in more accurate studies. For example, if the vehicle velocity is such that the maximum power coefficient imposed by the stoichiometric ratio (eq. (35)) is much less than the optimum power coefficient, the over-all range efficiency factor may have a very low value as can be seen in figure 4. In such cases high energy fuels which can increase the maximum power coefficient are of interest, even if the heat content per unit mass of the fuel is lower than it is for gasoline. Also, it is expected that external heat addition will provide large gains in such cases because of the use for combustion of otherwise unused air resulting in an increase of the effective power coefficient. In general, if the value of additional lift due to external combustion times vehicle velocity divided by the external heat power exceeds the over-all range efficiency factor $(LV/Q)_{T=0}$, the external combustion will increase the range. The linear theories of references 4 to 7 indicate a value of this ratio for external heat addition given by

$$\frac{L_Q V}{Q} = (\gamma - 1)\frac{M^2}{\sqrt{M^2 - 1}}$$

which, at Mach numbers greater than about 5, exceeds the typical value of $(LV/Q)_{T=0}$ equal to 2.0 for ram jets, however, the experimental results of references 22 and 24 indicate a value of $L_Q V/Q$ which is about one half the value predicted by linear theory at the Mach numbers of 2.5 and 3.0 of the tests. Also, exact numerical calculations in reference 8 indicate the linear theory results to be high by about a factor of 2. These results lead to the prediction that underwing heat addition will not increase the range at Mach numbers as low as 5.

In reference 7, methods for possible further improvement of the range efficiency utilizing external heat addition are treated. In a later section of this report methods for evaluating the effect on range of external heat addition are developed in more detail.

Burnout Velocity Mission

The efficiency ratio for the burnout velocity mission can be written with the aid of equations (19) and (20) as

$$\left(\frac{TV}{Q}\right)_{L=W} = 2\eta_t\left\{\frac{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}] - C_D(S/A)}{(\tilde{V}_e/V)^2 - \tilde{\eta}_k}\right\}_{C_L = C_W} \tag{38}$$

where C_L is a total lift coefficient given by the relation

$$C_L = \frac{L}{(1/2)\rho V^2 A} = 2\frac{\tilde{V}_e}{V}\sin\theta + \frac{L}{D}C_D\frac{S}{A} \tag{39}$$

C_W is a total weight coefficient defined as

$$C_W = \frac{W}{(1/2)\rho V^2 A} \tag{40}$$

and W is the instantaneous total airplane weight.

As before, S/A can be eliminated from equation (38) by virtue of the condition $C_L = C_W$, and the resulting expression maximized with respect to θ and $\tilde{V}_e/V$ with the following results:

$$\frac{S}{A} = \frac{2[(1/2)C_W - (\tilde{V}_e/V)\sin\theta]}{C_L} \tag{41}$$

$$\left(\frac{TV}{Q}\right)_{L=W} = 2\eta_t\left\{\frac{(\tilde{V}_e/V)\cos\theta-[1+(1/2)\tilde{C}_{D_{eng}}]-[1/(L/D)][(1/2)C_W-(\tilde{V}_e/V)\sin\theta]}{(\tilde{V}_e/V)^2-\tilde{\eta}_k}\right\}_{L=W} \tag{42}$$

$$\tan\theta_{opt} = \frac{1}{L/D} \tag{43}$$

$$\left(\frac{\tilde{V}_e}{V}\right)_{opt} = \sqrt{\tilde{\eta}_k}\left(\frac{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}} + \sqrt{\left\{\frac{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}}\right\}^2 - 1}\right) \tag{44}$$

$$\left(\frac{TV}{Q}\right)_{L=W_{opt}} = \frac{\eta_t\sqrt{1+(L/D)^2}/\sqrt{\tilde{\eta}_k}(L/D)}{\left\{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W/\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}\right\}+\sqrt{\left\{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W/\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}\right\}^2-1}} \tag{45}$$

Equations (43), (44), and (45) are similar to equations (28), (30), and (31) which apply to the range mission rather than the burnout velocity mission here under consideration. The main difference is that the loss parameter $\left\{[1 + (1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W\right\}/\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2} = x_{BV}$ has the additional term $(1/2)C_W/\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}$.

It should be noted that in practice the weight coefficient defined in equation (40) would depend explicitly on the ratio of wing size to engine size S/A, a factor which has been neglected in the derivation of equation (45). This dependence can be neglected if the variations in wing weight to be considered are a small enough fraction of the total airplane weight.

The ratio of wing size to engine size, the vehicle acceleration, and the power coefficient corresponding to the optimum efficiency represented by equation (45) are given by the relations

$$\left(\frac{S}{A}\right)_{opt} = \frac{2\sqrt{\tilde{\eta}_k}}{\sqrt{1 + (L/D)^2}\,C_L}\left[\frac{(1/2)\sqrt{1 + (L/D)^2}\,C_W}{\sqrt{\tilde{\eta}_k}} - x_{BV} - \sqrt{x_{BV}^2 - 1}\right] \tag{46}$$

$$\frac{(dV/dt)_{opt}}{g} = \frac{C_{T_{opt}}}{C_W} = 2\sqrt{\tilde{\eta}_k}\,\frac{\sqrt{1 + (L/D)^2}}{L/D}\sqrt{x_{BV}^2 - 1} \tag{47}$$

$$C_{Q_{opt}} = \frac{2\tilde{\eta}_k}{\eta_t}\sqrt{x_{BV}^2 - 1}\left(x_{BV} + \sqrt{x_{BV}^2 - 1}\right) \tag{48}$$

If low values of weight coefficient are contemplated $C_W \lesssim 1/3$, the optimum value of S/A is 0, and results different than those given above can be derived from equation (38) for the case of no wing.

If the optimum value of C_Q cannot be achieved because of the limit imposed by the stoichiometric ratio (eq. (35)), the dependence of efficiency on C_Q is of interest. This dependence is given by the relation

$$\left(\frac{TV}{Q}\right)_{L=W} = \frac{2\eta_t}{\sqrt{\tilde{\eta}_k}}\,\frac{\sqrt{1 + (L/D)^2}}{L/D}\left[\frac{\sqrt{1 + (\eta_t/\tilde{\eta}_k)C_Q} - x_{BV}}{(\eta_t/\tilde{\eta}_k)C_Q}\right] \tag{49}$$

The corresponding values of S/A are given by the expression

$$\frac{S}{A} = \frac{2\sqrt{\tilde{\eta}_k}}{\sqrt{1 + (L/D)^2}\,C_L}\left[\frac{(1/2)\sqrt{1 + (L/D)^2}\,C_W}{\sqrt{\tilde{\eta}_k}} - \sqrt{1 + \frac{\eta_t}{\tilde{\eta}_k}C_Q}\right] \tag{50}$$

Figure 6 is a plot of the reduced over-all airplane efficiency $[\sqrt{\tilde{\eta}_k}(L/D)/\eta_t\sqrt{1 + (L/D)^2}](TV/Q)_{L=W}$ versus the loss parameter for several values of the reduced power coefficient $(\eta_t/\tilde{\eta}_k)C_Q$, including the optimum value.

Figure 7 is a plot of the reduced value of the ratio of the wing plan-form area to engine inlet capture area $[\sqrt{1 + (L/D)^2}\,C_L/\sqrt{\tilde{\eta}_k}](S/A)$ with the parameter $\sqrt{1 + (L/D)^2}\,C_W/\sqrt{\tilde{\eta}_k}$ equal to 18 and for several values of the reduced power coefficient $(\eta_t/\tilde{\eta}_k)C_Q$, including the optimum value.

It can be seen in figure 6 that if there is an upper limit on the power coefficient which decreases with increasing vehicle speed as in equation (35), then there will be a maximum vehicle speed which occurs when $(TV/Q)_{L=W}$ is 0. Also there will be a maximum vehicle speed beyond which any given nonzero value of $(TV/Q)_{L=W}$ cannot be maintained. This maximum velocity is obtained by substituting equation (35) into (49) with the following results

$$\frac{V_{max}}{V_{sat}} = \sqrt{\frac{\alpha\eta_c\eta_t}{\tilde{\eta}_k}} \frac{b/\sqrt{2}}{\sqrt{1 - bx_{BV} - \sqrt{1 + b^2 - 2bx_{BV}}}} \tag{51}$$

$$b = \frac{\sqrt{\tilde{\eta}_k}(L/D)}{\eta_t\sqrt{1 + (L/D)^2}} \left(\frac{TV}{Q}\right)_{L=W}$$

Figure 8 is a plot of the quantity

$$\frac{1}{5}\sqrt{\frac{\tilde{\eta}_k}{\alpha\eta_c\eta_t}} \frac{V_{max}}{V_{sat}}$$

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as a function of the loss parameter for several values of the quantity

$$\frac{1}{2} \frac{\sqrt{\tilde{\eta}_k}(L/D)}{\eta_t\sqrt{1 + (L/D)^2}} \left(\frac{TV}{Q}\right)_{L=W}$$

For typical values of the parameters, the multiplication factors $(1/5)(\sqrt{\tilde{\eta}_k/\alpha\eta_c\eta_t})$ and $(1/2)[\sqrt{\tilde{\eta}_k}(L/D)/\eta_t\sqrt{1 + (L/D)^2}]$ are equal to 1, so that figure 8 can be taken to be a plot of V_{max}/V_{sat} versus the loss parameter for several values of $(TV/Q)_{L=W}$.

It should be emphasized that these are maximum velocities beyond which given values of over-all airplane efficiency (or vehicle acceleration efficiency) $(TV/Q)_{L=W}$ cannot be maintained, rather than maximum velocities which can be reached with some assumed engine efficiency. The curve for $(TV/Q)_{L=W}$ equal to 0 does have the special significance of representing the actual maximum vehicle speed because when T becomes 0 no further acceleration is possible.

The results plotted in figure 8 apply to turbojet airplanes as well as ram jets. Let us consider the requirements for increasing the maximum speed of a hypothetical aircraft represented by the $(TV/Q)_{L=W} = 0$ curve of figure 8. One way of increasing the maximum speed is to reduce the value of the loss parameter

$$\frac{[1 + (1/2)\tilde{C}_{D_{eng}}](L/D) + (1/2)C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}}$$

From the first term it might appear that reducing L/D would accomplish this. However, as previously mentioned, there is a lower limit on C_W

below which the optimum wing size is 0, and the results given here do not apply. For values of C_W above this limit, an increase of L/D will decrease the loss parameter as one would expect.

It can be seen qualitatively that the other obvious possibilities for increasing the maximum speed correspond to decreasing the value of the loss parameter.

Another possibility for increasing the maximum velocities indicated in figure 8 is by means of combustion under the wing. This is possible by virtue of the use for combustion of otherwise unused air resulting in an increase of the effective power coefficient. To increase the maximum vehicle velocity by means of external heat addition, it is not necessary to produce thrust, since production of lift by this means can decrease the thrust required to overcome drag due to lift.

To estimate the performance gains which can be achieved by external combustion, the appropriate terms can be added to equations (18), (19), and (20). The efficiency ratios $(LV/Q)_{T=0}$ and $(TV/Q)_{L=W}$ can be formed from the resulting expressions and maximized with respect to the design variables as before. The results of this procedure based on the linear theories of references 4 to 7 indicate that in those cases where external heat addition is advantageous, the amount of such heating should be as large as possible.

Methods for evaluating the effect on burnout velocity of external heat addition are treated more specifically in the next section.

Evaluation of the Effect on Performance of Heat Addition Under a Wing

In order to estimate the effect of external heat addition on performance, it will be assumed that such heating affects the wing lift and drag, but has no direct effect on the other airplane parameters. It will also be assumed at first that the heat content and combustion efficiency of engine and wing fuels are the same. Thus equations (18), (19), and (20) become the expressions

$$C_L = \frac{L}{(1/2)\rho V^2 A} = 2\,\frac{\tilde{V}_e}{V}\sin\theta + \frac{L}{D}\,C_D\,\frac{S}{A} \tag{52}$$

$$C_T = \frac{T}{(1/2)\rho V^2 A} = 2\left[\frac{\tilde{V}_e}{V}\cos\theta - \left(1 + \frac{1}{2}\,\tilde{C}_{D_{eng}}\right)\right] - C_D\,\frac{S}{A} \tag{53}$$

$$C_Q = \frac{Q}{(1/2)\rho V^2 A V} = \frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right] + C_{Q_{wing}}\,\frac{S}{A} \tag{54}$$

Equations (52) and (53) are formally identical to equations (18) and (19). However L/D and C_D are now functions of $C_{Q_{wing}}$, the wing power coefficient based on wing plan-form area. The explicit dependence will be discussed later.

Equations (52) through (54) can be used to write the efficiency ratio for the range mission. When this is done a certain grouping of terms leads to the definition of a quantity $(\lambda/\delta)_R$ which can be shown to be an effective lift to drag ratio. The relation

$$\left(\frac{LV}{Q}\right)_{T=0} = 2\eta_t\left(\frac{(\tilde{V}_e/V)\sin\theta + (\lambda/\delta)_R\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\}}{(\tilde{V}_e/V)^2 - \tilde{\eta}_k}\right) \tag{55}$$

applies, provided the ratio of wing size to engine size conforms to the relation

$$\frac{S}{A} = \frac{2\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\}}{C_D}$$

The quantity $(\lambda/\delta)_R$ appearing in equation (55) is defined by the expression

$$\left(\frac{\lambda}{\delta}\right)_R = \frac{C_L - \{2(\tilde{V}_e/V)\sin\theta/(1/\eta_t)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]\}C_{Q_{wing}}}{C_D + \left(2\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\}/(1/\eta_t)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]\right)C_{Q_{wing}}} \tag{56}$$

In general $(\lambda/\delta)_R$ is a function of all the design variables (S/A, $\tilde{V}_e/V$, θ, $C_{Q_{wing}}$). However, it is convenient to consider only those variations of the design variables for which $(\lambda/\delta)_R$ remains constant. Within the framework of the present model, it turns out that there is no loss of generality from this restriction.

If $(\lambda/\delta)_R$ is a constant, equation (55) is identical to equation (26) (no external heat addition) except that L/D is replaced by $(\lambda/\delta)_R$. Further, the restriction represented by equation (56) can be imposed with no restriction on the variables $\tilde{V}_e/V$, S/A, and θ by varying the quantity $C_{Q_{wing}}$ properly. Consequently, the external heat addition case reduces to the same problem as for no external heat addition previously analyzed. In this way the effect of external heat addition on range can be evaluated in terms of its effect on an equivalent lift to drag ratio represented by the quantity $(\lambda/\delta)_R$ given in equation (56).

It can be seen in equation (56) that if the wing power coefficient $C_{Q_{wing}}$ is 0, $(\lambda/\delta)_R$ is equal to L/D. In general, C_L and C_D depend on the value of $C_{Q_{wing}}$ in a way which can be determined independently of the variables $\tilde{V}_e/V$, θ, and S/A. However, some knowledge of the values of these variables is needed to evaluate $(\lambda/\delta)_R$. For this purpose, equation (56) can be rearranged to the following form:

$$\left(\frac{\lambda}{\delta}\right)_R = \frac{L}{D}\left\{\frac{1 - [(LV/Q)_{eng}/(LV/Q)_{wing}]}{1 + [(TV/Q)_{eng}(L/D)/(LV/Q)_{wing}]}\right\} \tag{57}$$

where the subscripts eng and wing refer to engine and wing quantities, respectively. Also L/D refers to the wing. Thus the quantity $(LV/Q)_{eng}$ is the product of lift due to engine exhaust deflection times vehicle velocity divided by the engine power, while the quantity $(LV/Q)_{wing}$ is the product of wing lift times vehicle velocity divided by the power expended in combustion under the wing. If no engine exhaust deflection is contemplated, the ratio $(LV/Q)_{eng}/(LV/Q)_{wing}$ is 0. The quantities L/D and $(LV/Q)_{wing}$ can be determined from experiments or calculations on the effects of combustion under wings. For many purposes, the quantities $(LV/Q)_{eng}$ and $(TV/Q)_{eng}$ can be estimated to be 0 and 1/3, respectively. However, at velocities near the maximum vehicle velocity, the quantity $(TV/Q)_{eng}$ tends to become small because of the limit on the power coefficient imposed by the stoichiometric ratio. Also the equivalent value of $(TV/Q)_{eng}$ for rocket motors, which can be calculated from the specific impulse and fuel heat content parameters, tends to be less than 1/3 at velocities below half of satellite speed. In the latter two cases $(\lambda/\delta)_R$ would be nearly equal to the full value of L/D which can be obtained by combustion under the wing.

In cases where $(\lambda/\delta)_R$ is not equal to L/D, it is apparent that the former should be maximized, rather than the latter. In these cases the optimum lift coefficient will be different in the presence of external combustion than in the absence of it. This point will be considered further after derivation of relations applicable to the burnout velocity mission similar to the preceding expressions for the range mission.

From equations (52) to (54), the burnout velocity efficiency ratio can be written as the relation

$$\left(\frac{TV}{Q}\right)_{L=W} = 2\eta_t\left\{\frac{(\tilde{V}_e/V)\cos\theta-[1+(1/2)\tilde{C}_{D_{eng}}]-[1/(\lambda/\delta)_{BV}][(1/2)C_W-(\tilde{V}_e/V)\sin\theta]}{(\tilde{V}_e/V)^2-\tilde{\eta}_k}\right\} \tag{58}$$

provided that

$$\frac{S}{A} = \frac{2[(1/2)C_W - (\tilde{V}_e/V)\sin\theta]}{C_L} \tag{59}$$

and

$$\left(\frac{\lambda}{\delta}\right)_{BV} = \frac{C_L + \left\{2[(1/2)C_W - (\tilde{V}_e/V)\sin\theta]/(1/\eta_t)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]\right\}C_{Q_{wing}}}{C_D + \left(2\left\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\right\}/(1/\eta_t)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]\right)C_{Q_{wing}}} \tag{60}$$

Again if $C_{Q_{wing}}$ is varied so as to hold $(\lambda/\delta)_{BV}$ constant, the problem reduces to the case of no external combustion treated previously, but with L/D replaced by $(\lambda/\delta)_{BV}$. Equation (60) can be rewritten, in this case as the relation

$$\left(\frac{\lambda}{\delta}\right)_{BV} = \frac{L}{D}\left\{\frac{1 + (Q_{wing}/Q_{eng})}{1 + [(TV/Q)_{eng}(L/D)/(LV/Q)_{wing}]}\right\} \tag{61}$$

or alternatively

$$\left(\frac{\lambda}{\delta}\right)_{BV} = \frac{L}{D}\left[\frac{1 + (Q_{wing}/Q_{eng})}{1 + (T_{eng}/D)(Q_{wing}/Q_{eng})}\right] \tag{62}$$

During acceleration, the engine thrust T_{eng} must exceed the wing drag D. Consequently from equation (62) it is seen that the effective lift to drag ratio $(\lambda/\delta)_{BV}$ is less than the actual lift to drag ratio. However, the vehicle maximum velocity is determined by the condition of engine thrust equal to wing drag, so that at velocities near this limit, almost the full value of L/D would be realized. This means that the vehicle maximum velocity can be increased by external combustion. The amount of the velocity increase can be estimated from figure 8 or equation (51) when the design parameters are known and the increase in L/D due to external combustion is known.

For example consider a hypothetical turbojet interceptor airplane which under maximum speed conditions has a value of the loss parameter $x_{BV} = 2.5$. If it is assumed that the typical values of 1 cited for figure 8 apply, the maximum speed indicated by figure 8 or equation (51) is 2280 feet per second (the maximum speed occurs at $(TV/Q)_{L=W} = 0$). If it is further assumed that the term $(1/2)C_W/\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}$ is equal to 0.6, and that enough heat can be added under the wing to approximately double the value of L/D, the loss parameter can be reduced to a value of 2.2. From figure 8 or equation (51), the maximum speed would then be estimated as 2650 feet per second, an increase of 370 feet per second (16 percent).

For a ram-jet configuration, where smaller values of the loss parameter are possible, larger percentage gains would result from doubling the L/D because of the steepening of the curves for small values of the loss parameter indicated in figure 8.

In the foregoing example nothing has been said about efficiency. In this application, the underwing heat addition does not necessarily increase the efficiency, but it does increase the maximum speed, for the same reason that afterburning increases the speed, namely because the total power is increased.

To determine the effect of external combustion on efficiencies, it is necessary to know the power required to produce the increase in L/D. Comparison of equations (57) and (62) shows that in the presence of external combustion, the value of the effective lift to drag ratio depends on the particular mission under consideration.

It should be emphasized that the effective lift to drag ratios represented by equations (57) and (62) were derived with few assumptions. These relations are not restricted to linearized theory, and should be applicable to rather general experimental situations.

In order to illustrate the method appropriate for optimizing the wing lift coefficient in the presence of external combustion, a simplified model corresponding to that used in reference 24 and elsewhere will be adopted as follows:

$$C_L = C_{L_a} + \frac{L_Q V}{Q} C_Q \tag{63}$$

$$C_D = C_{D_o} + \frac{dC_D}{dC_{L_a}{}^2}\left(C_{L_a}{}^2 + \frac{L_Q V}{Q} C_{L_a} C_Q\right) - \frac{C_{T_o}}{C_Q} C_Q \tag{64}$$

where

C_{L_a} part of the lift coefficient due to angle of attack and camber

L_Q additional lift due to external heat addition

Q external heat power

C_Q external power coefficient based on wing plan-form area

C_{D_o} drag coefficient at zero lift coefficient and zero external power coefficient

C_{T_o} thrust coefficient due to heat addition at zero lift coefficient

In these relations, all quantities are wing quantities and the wing subscript has been omitted. In the subsequent development, quantities without the engine subscript are wing quantities.

With the definitions given, equation (63) is an identity. In general, the quantity L_QV/Q would depend on C_{L_a} and C_Q. However, it will be assumed that L_QV/Q is a constant in accordance with the linear theories of references 3 to 7. Similarly, equation (64) can be derived from linear theory, and in that case the quantities $dC_D/dC_{L_a}{}^2$ and C_{T_o}/C_Q are constant if the magnitude of camber is held proportional to C_{L_a} and the thickness ratio is held constant.

We wish to maximize $(\lambda/\delta)_R$ and $(\lambda/\delta)_{BV}$ with respect to C_{L_a} and also with respect to C_Q. For this purpose, equations (63) and (64) can be used to write equation (56) as the relation

$$\left(\frac{\lambda}{\delta}\right)_R = \frac{C_{L_a}+[(L_QV/Q)-(LV/Q)_{eng}]C_Q}{C_{D_o}+(dC_D/dC_{L_a}{}^2)C_{L_a}{}^2+[(dC_D/dC_{L_a}{}^2)(L_QV/Q)C_{L_a}-(C_{T_o}/C_Q)+(TV/Q)_{eng}]C_Q} \tag{65}$$

All quantities appearing here are to be considered constant except C_{L_a} and C_Q. In that case $(\lambda/\delta)_R$ will be a maximum for either a zero or an infinite value of C_Q, depending on whether or not the quantity $C_{L_a}/[C_{D_o} + (dC_D/dC_{L_a}{}^2)C_{L_a}{}^2]$ exceeds the quantity

$$\frac{(L_QV/Q) - (LV/Q)_{eng}}{(dC_D/dC_{L_a}{}^2)(L_QV/Q)C_{L_a} - (C_{T_o}/C_Q) + (TV/Q)_{eng}}$$

In the former case the result of NACA TN 1350 (ref. 24) is obtained. Otherwise C_Q should be made as large as possible, and equation (65) maximized with respect to C_{L_a} for a fixed value of C_Q. For this purpose, it is convenient to rewrite equation (65) in terms of the total lift coefficient C_L, and find the optimum value of this quantity for a fixed value of C_Q as follows:

$$\left(\frac{\lambda}{\delta}\right)_R = \frac{C_L-(LV/Q)_{eng}C_Q}{C_{D_o}+[(TV/Q)_{eng}-(C_{T_o}/C_Q)]C_Q+(dC_D/dC_{L_a}{}^2)C_L{}^2-(dC_D/dC_{L_a}{}^2)(L_QV/Q)C_QC_L} \tag{66}$$

By differentiation, it is found that the optimum values of $(\lambda/\delta)_R$ and C_L are given by the relations

$$\left(\frac{\lambda}{\delta}\right)_{R_{opt}} = \frac{1}{2(dC_D/dC_{L_a}{}^2)[C_{L_{opt}} - (1/2)(L_QV/Q)C_Q]} \tag{67}$$

$$C_{L_{opt}} = \sqrt{\frac{C_{D_o} + [(TV/Q)_{eng} - (C_{T_o}/C_Q)]C_Q}{dC_D/dC_{L_a}{}^2} - \left[\frac{LQV}{Q} - \left(\frac{LV}{Q}\right)_{eng}\right]\left(\frac{LV}{Q}\right)_{eng} C_Q{}^2 + \left(\frac{LV}{Q}\right)_{eng} C_Q} \quad (68)$$

In the case of no heating ($C_Q = 0$), equations (67) and (68) reduce to the result of reference 24; namely

$$\left(\frac{\lambda}{\delta}\right)_{R_{opt}} = \frac{1}{2\sqrt{C_{D_o}(dC_D/dC_{L_a}{}^2)}} \quad (69)$$

$$C_{L_{opt}} = \sqrt{\frac{C_{D_o}}{dC_D/dC_{L_a}{}^2}} \quad (70)$$

It is interesting to note that if the value of C_Q is given, equations (67) and (68) must be evaluated by successive approximations also using equations (18) to (29). The reason for this is that $(LV/Q)_{eng}$ and $(TV/Q)_{eng}$ depend on the value of $(\lambda/\delta)_R$, which replaces L/D in the earlier relations.

On the other hand, if a value of $(\lambda/\delta)_R$ is given, the required value of C_Q can be determined from equation (67) in terms of $C_{L_{opt}}$, and $C_{L_{opt}}$ can be determined from equation (68) together with equations (18) to (29).

Starting with equation (55) the foregoing procedure is restricted by the requirement $C_D > 0$ where

$$C_D = C_{D_o} - \frac{C_{T_o}}{C_Q} C_Q + \frac{dC_D}{dC_{L_a}{}^2} C_L{}^2 - \frac{dC_D}{dC_{L_a}{}^2} \frac{LQV}{Q} C_Q C_L \quad (71)$$

This puts an upper limit on C_Q. Consequently $(\lambda/\delta)_R$ cannot exceed the value which yields an over-all value of $(LV/Q)_{T=0}$ equal to the wing quantity $(LV/Q)_{D=0}$. If a value of C_Q large enough to reach this limit is possible, no engine is needed, that is, $(S/A)_{opt}$ approaches infinity.

In the foregoing analysis, the wing thickness ratio has been held constant. In reference 7, it is shown that for given heating, and a given thickness distribution, there is an optimum nonzero thickness ratio because of the dependence of the quantity C_{T_o}/C_Q on the thickness ratio. When this factor is taken into account, values of $(LV/Q)_{D=0}$ greater than the value for a frictionless flat plate are possible according to the linear theory. Preliminary results of an investigation of this possibility using exact theory have failed to show the expected gains in the cases considered. However, all cases of interest have not been studied as yet.

In the burnout velocity case, underwing heat addition will be advantageous at lower speeds than in the range case, because of the added term in the loss parameter. Equations analogous to those for the range mission are obtained as follows:

$$\left(\frac{\lambda}{\delta}\right)_{BV} = \frac{C_{L_a}+[(L_QV/Q)-(LV/Q)_{eng}+(C_W/C_{Q_{eng}})]C_Q}{C_{D_o}+(dC_D/dC_{L_a}{}^2)C_{L_a}{}^2+[(dC_D/dC_{L_a}{}^2)(L_QV/Q)C_{L_a}-(C_{T_o}/C_Q)+(TV/Q)_{eng}]C_Q} \tag{72}$$

$$\left(\frac{\lambda}{\delta}\right)_{BV_{opt}} = \frac{1}{2(dC_D/dC_{L_a}{}^2)[C_{L_{opt}} - (1/2)(L_QV/Q)C_Q]} \tag{73}$$

$$C_{L_{opt}} = \sqrt{\frac{C_{D_o}+[(TV/Q)_{eng}-(C_{T_o}/C_Q)]C_Q}{dC_D/dC_{L_a}{}^2} + \left[\frac{C_W}{C_{Q_{eng}}} - \left(\frac{LV}{Q}\right)_{eng}\right]\left[\frac{L_QV}{Q} + \frac{C_W}{C_{Q_{eng}}} - \left(\frac{LV}{Q}\right)_{eng}\right]} - \left[\frac{C_W}{C_{Q_{eng}}} - \left(\frac{LV}{Q}\right)_{eng}\right]C_Q \tag{74}$$

Comparison of equations (73) and (74) with (67) and (68) shows that the optimum value of effective lift to drag ratio (represented by λ/δ) and the optimum lift coefficient depend not only on engine parameters but also on the mission under consideration.

All of the relations given in this section can be altered to take into account a possible difference in the wing and engine fuel heat content parameters (k). This is accomplished by dividing Q_{eng} and $C_{Q_{eng}}$ everywhere by k_{eng} and by dividing Q_{wing} and $C_{Q_{wing}}$ everywhere by k_{wing}. For example equations (57) and (62) become the relations

$$\left(\frac{\lambda}{\delta}\right)_R = \frac{L}{D}\left\{\frac{1 - [(k_{eng}/k_{wing})(LV/Q)_{eng}/(LV/Q)_{wing}]}{1 + [(k_{eng}/k_{wing})(TV/Q)_{eng}(L/D)/(LV/Q)_{wing}]}\right\} \tag{75}$$

$$\left(\frac{\lambda}{\delta}\right)_{BV} = \frac{L}{D}\left[\frac{1 + (k_{eng}/k_{wing})(Q_{wing}/Q_{eng})}{1 + (T_{eng}/D_{wing})(k_{eng}/k_{wing})(Q_{wing}/Q_{eng})}\right] \tag{76}$$

Thus if the wing fuel has lower heat content per unit weight than the engine fuel, the effective lift to drag ratio λ/δ will be less than if the heat content were equal.

Equations (75) and (76) can be further modified to take into account unequal combustion efficiencies by multiplying each heat content parameter by a corresponding combustion efficiency.

Fuselage Drag

In applying the foregoing results to airplanes, the question arises as to whether the fuselage drag should be included as engine drag or as wing drag, assuming that these items can be separately defined. It is apparent that the same effect will not result from both choices. For example, if an additional drag is included as wing drag, the optimum lift coefficient will be increased. If the same drag is charged instead to the engine, the optimum power coefficient will be increased, but the optimum lift coefficient will be unaffected by the addition.

Whether the fuselage drag should be charged to the engine or to the wing depends on whether the fuselage size is held proportional to the engine size or to the wing size when a variation of relative sizes is considered. In either case, it is desirable to hold the fuselage drag to as low a value as possible.

At hypersonic speeds, it may be possible to hold this drag to a low level by placing the fuselage in the low pressure region above the wing, or by using it to produce lift, such that the wing-fuselage combination is as efficient as a wing alone would be. If a large enough fuselage can be obtained in this way, the fuselage drag can be charged to the wing. However, when the fuselage drag is a large adverse item, it should probably be charged to the engine in analyzing the range and burnout velocity missions. Justification for this contention is given in appendix C. As a result, the optimum power coefficient may be considerably increased over the value which would apply to the engine alone.

Rocket Ram Jet

Several types of hypersonic vehicles should be analyzed for comparison with configurations employing external heat addition. One of these, the rocket ram jet is essentially composed of a rocket motor exhausting into the combustion chamber of a ram jet (see ref. 25). Such an arrangement has the advantage over an ordinary ram jet that it can produce thrust at all velocities from zero to beyond the maximum velocity of the ordinary ram jet. At low speeds the specific impulse of the rocket motor is augmented by virtue of the increased jet efficiency resulting from the mixing of air with the rocket exhaust. At intermediate speeds where the ram jet is most efficient, the rocket fuel can be conserved. At high speeds where the ram jet optimum power coefficient exceeds the value imposed by the stoichiometric ratio, the rocket motor can be used to increase the effective power coefficient. These effects can be approximated by considering a conventional ram jet which uses two independent fuels. In that case, it is the quantity $k(TV/Q)_{L=W}$ appearing in equation (9) which should be maximized rather that $(TV/Q)_{L=W}$ alone, since the effective value of k depends on the relative values of the mass flow for the two fuels. If for example gasoline is considered as one fuel and the combination of gasoline and liquid oxygen as the second fuel, the heat content per unit mass of the second fuel is about one third the value for the first fuel. At vehicle speeds greater than those for which the optimum power coefficient can be achieved by the first fuel alone, there will be a nonzero optimum value of mass flow for the second fuel. This optimum for the burnout velocity mission can be obtained by expressing the combination $k(TV/Q)_{L=W}$ as follows:

$$k\left(\frac{TV}{Q}\right)_{L=W} = \left[\frac{TV}{(Q_1/k_1) + (Q_2/k_2)}\right]_{L=W} = k_2\left[\frac{TV}{(k_2/k_1)Q_1 + Q_2}\right]_{L=W} \tag{77}$$

where the subscript (1) refers to the first fuel and the subscript (2) to the second fuel. Since the quantity $Q_1 + Q_2$ is the total heat added per unit time, equation (20) is replaced by

$$Q_1 + Q_2 = \frac{1}{2}\rho V^2 AV\left\{\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right]\right\} \tag{78}$$

while equations (18) and (19) remain the same.

Substitution of equations (19) and (77) into (78) yields

$$\frac{k}{k_1}\left(\frac{TV}{Q}\right)_{L=W} = \frac{k_2}{k_1}\left(\frac{2\left\{(\tilde{V}_e/V)\cos\theta - [1 + (1/2)\tilde{C}_{D_{eng}}]\right\}C_D(S/A)}{(1/\eta_t)\left\{(\tilde{V}_e/V)^2 - \tilde{\eta}_k - [1 + (k_2/k_1)]\eta_t C_{Q_1}\right\}}\right)_{C_L = C_W} \tag{79}$$

We wish to maximize this expression with respect to θ and $\tilde{V}_e/V$ for a fixed value of C_{Q_1}. This process is formally the same as the process following equation (38), since equation (79) differs from (38) only in the constant factor k_2/k_1 and in that the quantity $\tilde{\eta}_k+[1-(k_2/k_1)]\eta_t C_{Q_1}$ of equation (79) replaces the quantity $\tilde{\eta}_k$ in equation (38). The result is the relation

$$\left(\frac{k}{k_1}\frac{TV}{Q}\right)_{L=W_{opt}} = \frac{k_2/k_1}{\sqrt{1+[1-(k_2/k_1)](\eta_t/\eta_k)C_{Q_1}}}\,\frac{\eta_t\sqrt{1+(L/D)^2}}{\tilde{\eta}_k(L/D)}$$
$$\left(\frac{1}{\frac{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W}{\sqrt{1+[1-(k_2/k_1)](\eta_t/\tilde{\eta}_k)C_{Q_1}}\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}} + \sqrt{\left\{\frac{[1+(1/2)\tilde{C}_{D_{eng}}](L/D)+(1/2)C_W}{\sqrt{1+[1-(k_2/k_1)](\eta_t/\tilde{\eta}_k)C_{Q_1}}\sqrt{\tilde{\eta}_k}\sqrt{1+(L/D)^2}}\right\}^2 - 1}}\right) \tag{80}$$

This result differs from that for the single fuel ram jet mainly in the factor k_2/k_1, which for (1) gasoline, (2) liquid oxygen and gasoline is about 1/3. However this efficiency for the rocket ram jet can be maintained to a higher velocity than the ram jet can operate, assuming that the structural heating problems can be solved.

The efficiency represented by equation (80) corresponds to nearly perfect mixing efficiency and assumes recombination in the nozzle such that the exhaust gas is nearly in chemical equilibrium. On the other hand the additional thrust due to the fuel mass flow is also neglected, an unfavorable assumption which is not justified for large fuel mass flow.

Evolution of the rocket ram jet may result in the possibility of acceleration from rest to a velocity of 10,000 feet per second for a mass ratio of about 2. In such an application, external heat addition may prove to be important in reducing the amount of rocket fuel required.

CONCLUDING REMARKS

Basic ram-jet aircraft design considerations have been reviewed at a level of simplification appropriate for comparison of external heat addition schemes with conventional configurations.

The results of these studies can be used to determine whether external combustion is advantageous in comparison to more conventional arrangements when it is known what the forces resulting from combustion will be. Estimates of these forces vary widely in the speed range not covered by experimental results both for external combustion and for conventional ram jets. At Mach numbers 2.5 and 3.0 experimental results for combustion under a wing show the effect of such combustion on range to be disadvantageous compared to internal combustion. However, these results show that the maximum speed of existing airplanes could be increased by combustion under the wing in cases where the maximum speed is not determined by structural heating.

It is to be expected that friction drag and shock losses will continue to rise faster with increasing vehicle speed than does the air mass flow in conventional ram jets. Consequently if only air is available as an oxidant, at some Mach number between 5 and 10, the heat power available falls below the values required for efficient operation. In addition to external combustion, another possibility for counteracting this difficulty is the rocket ram jet which carries additional oxidant in order to increase the power available.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., May 26, 1959

APPENDIX A

SYMBOLS

A engine inlet capture area

A_e engine exhaust area

B fuselage frontal area

C_D wing drag coefficient, $\dfrac{D}{(1/2)\rho V^2 S}$

C_{D_B} fuselage drag coefficient, $\dfrac{D_B}{(1/2)\rho V^2 B}$

$C_{D_{eng}}$ engine drag coefficient, $\dfrac{D_{eng}}{(1/2)\rho V^2 A}$

$\tilde{C}_{D_{eng}}$ quantity, $C_{D_{eng}} + \dfrac{A_e}{A} \dfrac{p}{(1/2)\rho V^2}$

C_{D_o} wing drag coefficient at zero lift coefficient and zero wing power

C_L wing lift coefficient, $\dfrac{L}{(1/2)\rho V^2 S}$

C_{L_a} part of wing lift coefficient due to angle of attack and camber

C_L airplane lift coefficient, $\dfrac{L}{(1/2)\rho V^2 A}$

C_Q power coefficient, $\dfrac{Q}{(1/2)\rho V^2 AV}$

$C_{Q_{wing}}$ wing power coefficient, $\dfrac{Q_{wing}}{(1/2)\rho V^2 SV}$

C_{T_o} additional wing thrust due to heat addition at zero lift coefficient

C_T net airplane thrust coefficient, $\dfrac{T}{(1/2)\rho V^2 A}$

C_W airplane weight coefficient, $\dfrac{W}{(1/2)\rho V^2 A}$

D	wing drag (including drag due to combustion under wing when applicable)
D_B	fuselage drag
D_{eng}	engine drag
g	acceleration due to gravity, 32.2 ft/sec^2
h	free-stream enthalpy
h_e	enthalpy at the exit in the engine exhaust
I	effective specific impulse
k	dimensionless fuel heat content parameter defined in equations (6) and (7)
L	wing lift (including lift due to combustion under wing when applicable)
L_Q	additional wing lift due to combustion under the wing
L	total airplane lift
M	free-stream Mach number
p	free-stream pressure
p_e	pressure at the exit in the engine exhaust
Q	heat power added to the air
Q_{eng}	heat power added to the air in the engine
Q_{wing}	heat power added to the air under the wing
S	wing plan-form area
t	time
T_{eng}	engine propulsive thrust
T	over-all airplane thrust, engine thrust minus airplane drag
V	vehicle velocity or free-stream velocity
V_e	velocity at the exit in the engine exhaust

$\tilde{V}_e$ quantity, $V_e\left(1 + \frac{p_e}{\rho_e V_e^2}\right)$

V_{sat} satellite velocity, 26,100 ft/sec

W total instantaneous airplane weight

x_{BV} loss parameter equal to $\dfrac{[1 + (1/2)\tilde{C}_{D_{eng}}](L/D) + (1/2)C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}}$

x_R loss parameter equal to $\dfrac{[1 + (1/2)\tilde{C}_{D_{eng}}](L/D)}{\sqrt{\tilde{\eta}_k}\sqrt{1 + (L/D)^2}}$

α combustion parameter defined in equation (35)

γ ratio of specific heats for air, 7/5

η over-all airplane efficiency, $\left(\frac{TV}{Q}\right)_{L=W}$

η_c combustion efficiency

η_e engine over-all efficiency

η_k over-all kinetic energy efficiency

$\tilde{\eta}_k$ quantity, $\eta_k\left(1 + \frac{p}{\rho V^2}\right)^2$

η_t ideal thermodynamic cycle efficiency

$\frac{\lambda}{\delta}$ effective lift to drag ratio in the presence of external combustion

θ engine exhaust deflection from horizontal

ρ free-stream density

ρ_e density at the exit in the engine exhaust

Subscripts

An equation written as a subscript on a bracket denotes a condition imposed on the bracketed quantity

a	aerodynamic (e.g., C_{L_a}, the part of the lift coefficient due to angle of attack and camber)
BV	burnout velocity mission
e	conditions at the engine exhaust or exit plane
eng	engine quantities
i	initial conditions
max	maximum value
opt	optimum value
Q	heating (e.g., L_Q, the additional wing lift due to combustion under the wing)
R	range mission
wing	wing quantities
1	first fuel
2	second fuel

APPENDIX B

EVALUATION OF ENGINE PARAMETERS FROM EXPERIMENTAL DATA

If only engine quantities are retained, equations (19) and (20) become the relations

$$T = \frac{1}{2}\,\rho V^2 A\left\{2\left[\frac{\tilde{V}_e}{V} - \left(1 + \frac{1}{2}\,\tilde{C}_{D_{eng}}\right)\right]\right\} \tag{B1}$$

$$Q = \frac{1}{2}\,\rho V^2 AV\left\{\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right]\right\} \tag{B2}$$

Using these relations, we wish to evaluate the constants $\tilde{C}_{D_{eng}}$, η_t, and $\tilde{\eta}_k$ from experimental measurements of the thrust T at several values of the heat power Q. It may be recalled that the heat power is related to the fuel mass flow by the relation

$$Q = \eta_c k\,\frac{1}{2}\,V_{sat}^2\,\frac{d(-W/g)}{dt} \tag{6}$$

where $d(-W/g)/dt$ is the fuel mass flow, and the other quantities are defined in appendix A.

It should be recognized that the value of $\tilde{C}_{D_{eng}}$ which will be determined in this process will not necessarily be the same which will apply when the engine is installed in an airplane; $\tilde{C}_{D_{eng}}$ is defined by the relation

$$\tilde{C}_{D_{eng}} = C_{D_{eng}} + \frac{A_e}{A}\,\frac{p}{(1/2)\rho V^2} \tag{24}$$

where

$$C_{D_{eng}} = \frac{D_{eng}}{(1/2)\rho V^2 A} \tag{B3}$$

When the engine is installed in an airplane, the quantity D_{eng} should encompass an external drag including the fuselage drag as discussed in the text and in appendix C. Such terms, not included in the experimental data, can be added later.

Since we wish to establish equations (B1) and (B2) on a semiempirical basis, the basic component of $C_{D_{eng}}$ cannot be accurately evaluated independently of the other parameters η_t and $\tilde{\eta}_k$.

It is convenient to define thrust and power coefficients by the relations

$$C_T = \frac{T}{(1/2)\rho V^2 A} = 2\left[\frac{\tilde{V}_e}{V} - \left(1 + \frac{1}{2}\,\tilde{C}_{D_{eng}}\right)\right] \tag{B4}$$

$$C_Q = \frac{Q}{(1/2)\rho V^2 AV} = \frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right] \tag{B5}$$

Eliminating the quantity $\tilde{V}_e/V$ from these equations yields the relation

$$C_Q = \frac{1}{\eta_t}\left\{\left[\frac{1}{2}\,C_T + \left(1 + \frac{1}{2}\,\tilde{C}_{D_{eng}}\right)\right]^2 - \tilde{\eta}_k\right\}$$

or

$$\frac{\eta_t}{\tilde{\eta}_k}\,C_Q = \left\{\frac{C_T}{2\sqrt{\tilde{\eta}_k}} + \left[\frac{1 + (1/2)\tilde{C}_{D_{eng}}}{\sqrt{\tilde{\eta}_k}}\right]\right\}^2 - 1 \tag{B6}$$

Then we wish to determine η_t, $\tilde{\eta}_k$, and $\tilde{C}_{D_{eng}}$ by fitting the experimental data with a relation of the form

$$aC_Q = (bC_T + c)^2 - 1 \tag{B7}$$

where

$$a = \frac{\eta_t}{\tilde{\eta}_k} \tag{B8}$$

$$b = \frac{1}{2\sqrt{\tilde{\eta}_k}} \tag{B9}$$

$$c = \frac{1 + (1/2)\tilde{C}_{D_{eng}}}{\sqrt{\tilde{\eta}_k}} \tag{B10}$$

and

$$\eta_t = \frac{a}{4b^2} \tag{B11}$$

$$\tilde{\eta}_k = \frac{1}{4b^2} \tag{B12}$$

$$\tilde{C}_{D_{eng}} = \frac{c}{b} - 2 \tag{B13}$$

Engine data are not often given in the form of C_Q versus C_T, but are sometimes given as over-all engine efficiency η_e versus C_T. The over-all engine efficiency can be expressed in terms of C_Q and C_T by the relations

$$\frac{\eta_e}{\eta_c} = \frac{TV}{Q} = \frac{C_T}{C_Q} = \frac{a}{b} \frac{bC_T}{(bC_T + c)^2 - 1} \tag{B14}$$

where η_c is the combustion efficiency.

Equation (B14) can always be fitted to the experimental data in the neighborhood of a particular value of the thrust coefficient. In general, the values of the parameters η_t, $\tilde{\eta}_k$, and $\tilde{C}_{D_{eng}}$ determined in this way will vary with free-stream Mach number and pressure and will vary with C_T, but because of their direct connection with airplane performance, they are useful concepts.

Values of η_t determined in this way from calculated data for ram-jet engines, given in reference 23, vary between 0.53 and 0.64, while $\tilde{\eta}_k$ varies from 1.13 to 1.30, and $\tilde{C}_{D_{eng}}$ from 0.18 to 0.34. These calculations were for the Mach number range from 3 to 7.

APPENDIX C

ARGUMENT FOR INCLUDING FUSELAGE DRAG IN ENGINE DRAG TERM

Suppose we have a given fuselage with frontal area B. What size should the engine and wing be? To answer this question, equations (18), (19), and (20) can be rearranged as follows:

$$L = \frac{1}{2}\rho V^2 B\left(\frac{A}{B}\,2\,\frac{\tilde{V}_e}{V}\sin\theta + \frac{L}{D}C_D\frac{S}{B}\right) \tag{C1}$$

$$T = \frac{1}{2}\rho V^2 B\left\{\frac{A}{B}\,2\left[\frac{\tilde{V}_e}{V}\cos\theta - \left(1 + \frac{1}{2}\tilde{C}_{D_{eng}}\right)\right] - C_{D_B} - C_D\frac{S}{B}\right\} \tag{C2}$$

$$Q = \frac{1}{2}\rho V^2 BV\left\{\frac{A}{B}\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right]\right\} \tag{C3}$$

where

B fuselage frontal area (body)

A engine inlet capture area

S wing plan-form area

C_{D_B} fuselage drag coefficient

and the other quantities are as defined in the text.

The range efficiency factor can be written as

$$\left(\frac{LV}{Q}\right)_{T=0} = \left\{\frac{\eta_t[(A/B)2(\tilde{V}_e/V)\sin\theta + (L/D)(C_D)(S/B)]}{(A/B)[(\tilde{V}_e/V)^2 - \tilde{\eta}_k]}\right\}_{T=0} \tag{C4}$$

The condition $T = 0$ will be imposed if the relation

$$\frac{L}{D}C_D\frac{S}{B} = \frac{L}{D}\left\{\frac{A}{B}\,2\left[\frac{\tilde{V}_e}{V}\cos\theta - \left(1 + \frac{1}{2}\tilde{C}_{D_{eng}}\right)\right] - C_{D_B}\right\} \tag{C5}$$

is satisfied.

Substitution of equation (C5) into (C4) and rearrangement yields the relation

$$\left(\frac{LV}{Q}\right)_{T=0} = 2\eta_t\left(\frac{(\tilde{V}_e/V)\sin\theta+(L/D)\left\{(\tilde{V}_e/V)\cos\theta-[1+(1/2)\tilde{C}_{D_{eng}}+(1/2)(B/A)C_{D_B}]\right\}}{(\tilde{V}_e/V)^2-\tilde{\eta}_k}\right) \quad (C6)$$

This equation is identical to equation (26) of the text except that the quantity $1 + (1/2)\tilde{C}_{D_{eng}}$ in equation (26) is replaced by the quantity $1 + (1/2)\tilde{C}_{D_{eng}} + (1/2)(B/A)C_{D_B}$. If the wing size is to be varied so that equation (C5) does not prevent $\tilde{V}_e/V$ and θ from varying freely, it can be concluded that: (1) the ratio of fuselage size to engine size should be as small as possible; (2) the fuselage drag should be included in the engine drag term.

The same conclusions can be reached for the burnout velocity mission.

Equation (C1) does not include a fuselage lift term. Inclusion of such a term does not alter the conclusions as long as the fuselage lift to drag ratio is less than that of the wing. The fuselage optimum lift coefficient occurs at the maximum fuselage lift to drag ratio, as one would expect. When the fuselage develops lift, the fuselage drag is effectively decreased by the factor

$$1 - \frac{(L/D)_{fuselage}}{(L/D)_{wing}} \quad (C7)$$

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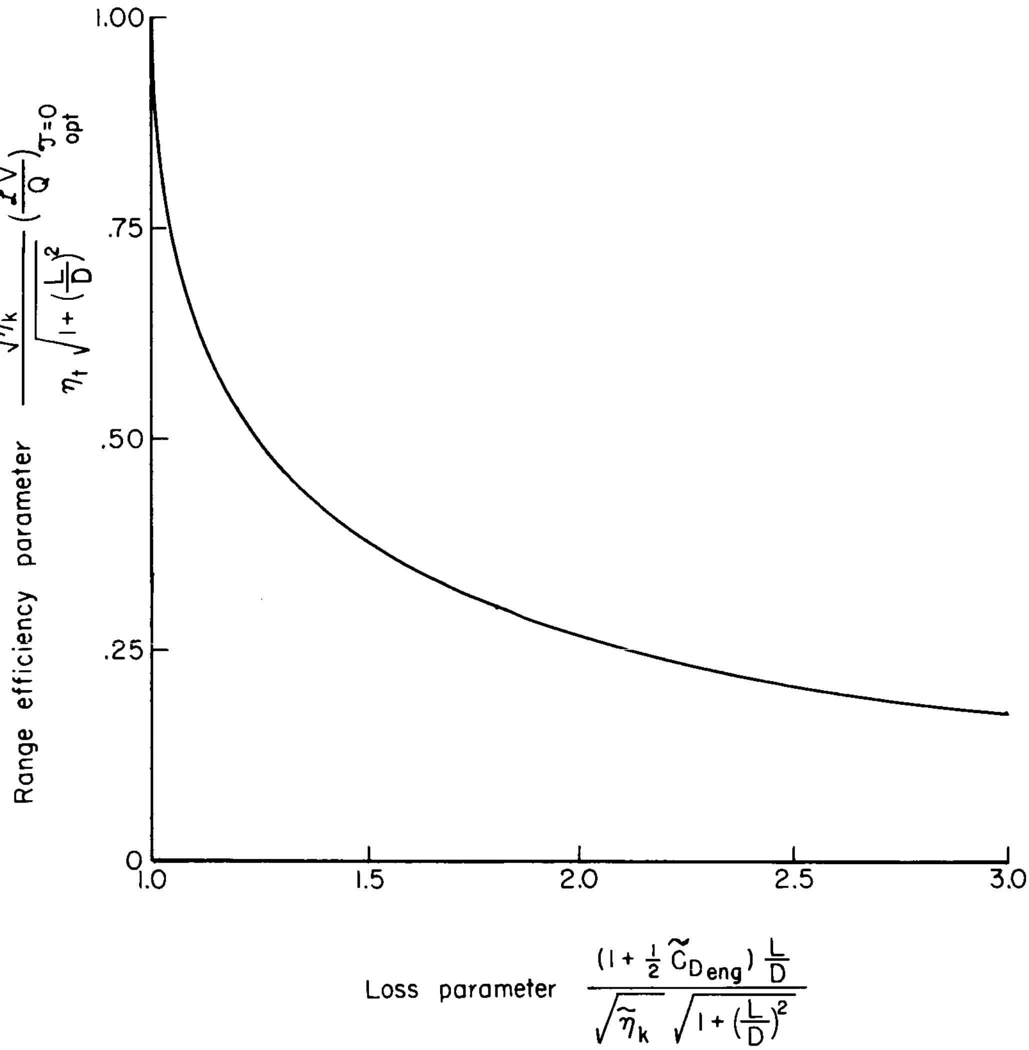

Figure 1.- Optimum values of range efficiency.

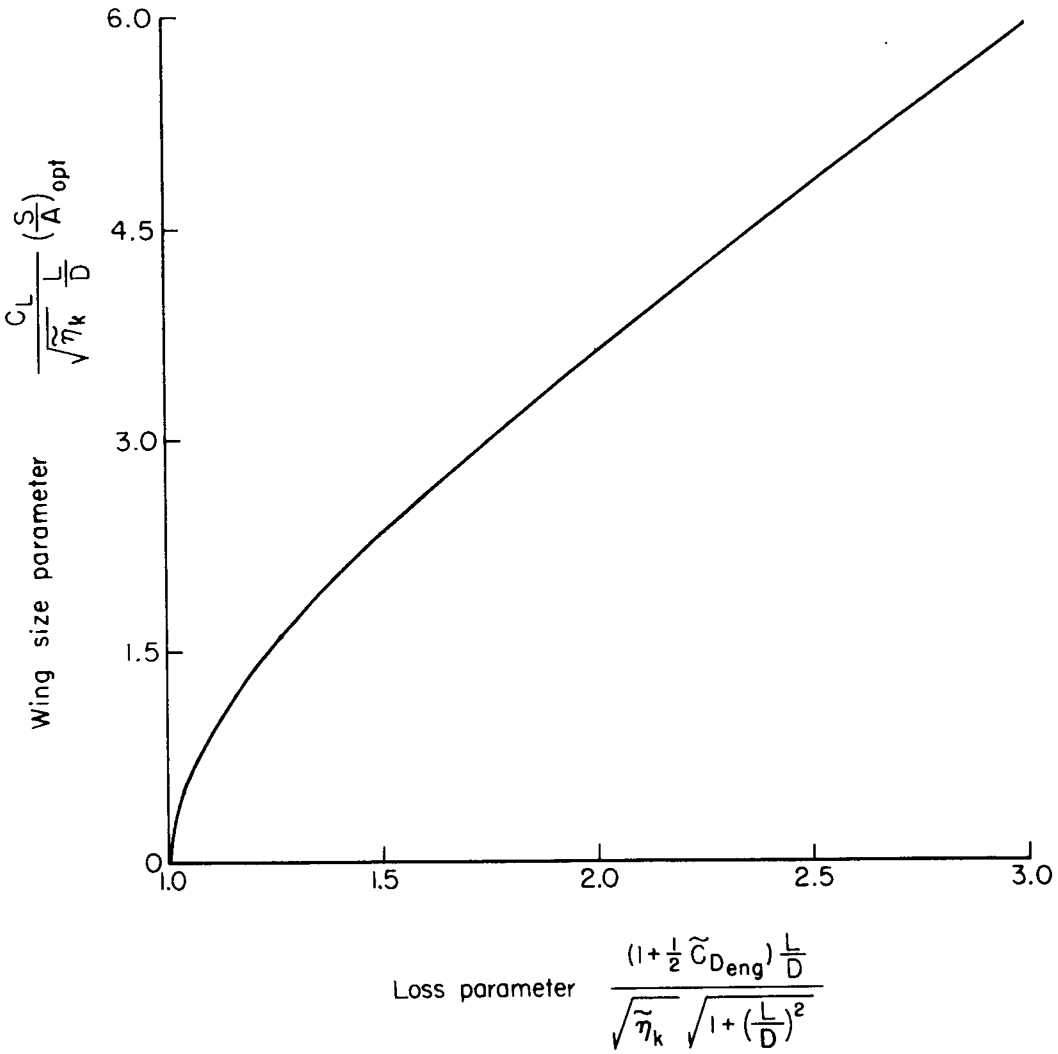

Figure 2.- Ratio of wing plan-form area to engine inlet capture area for maximum range.

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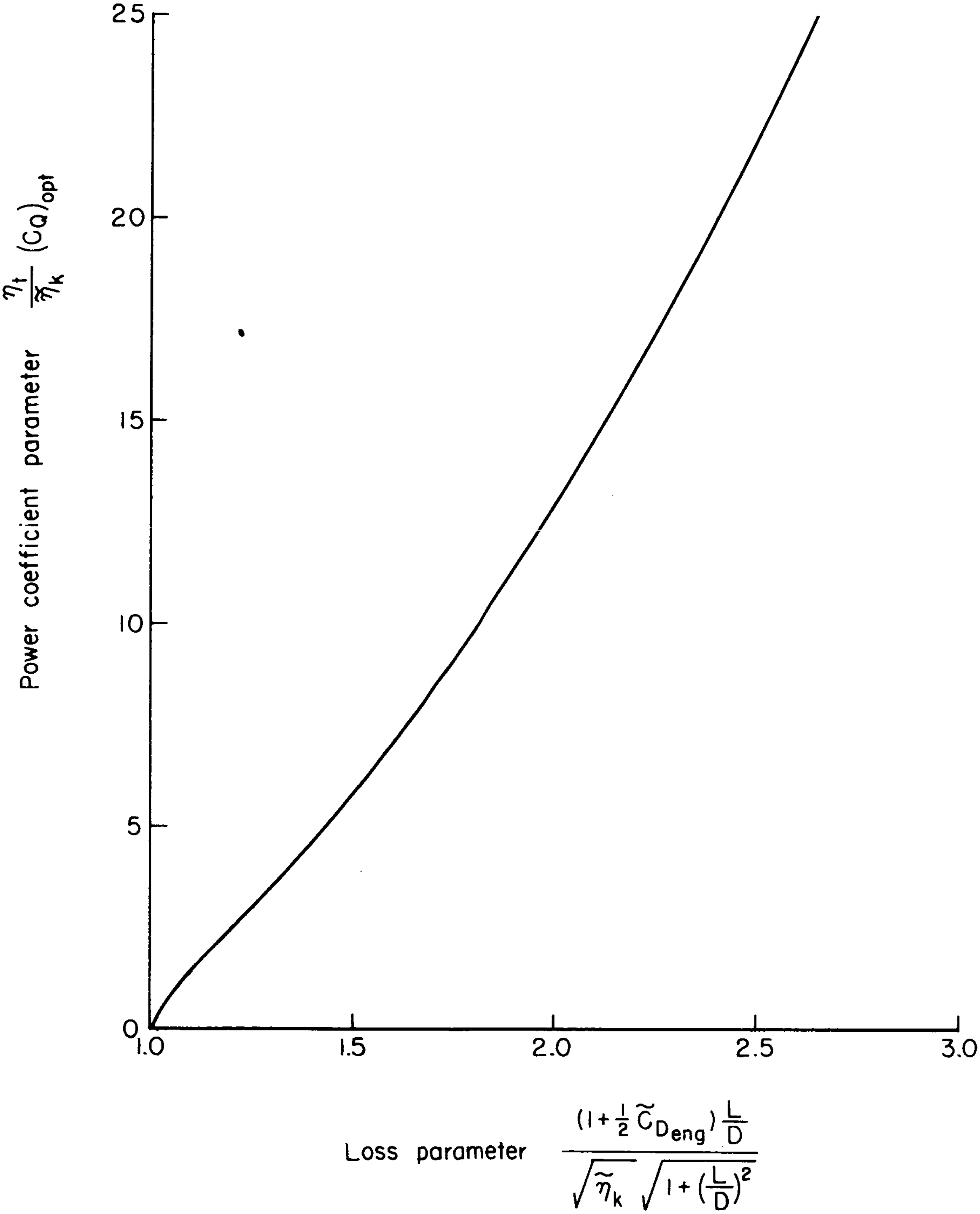

Figure 3.- Power coefficient for maximum range.

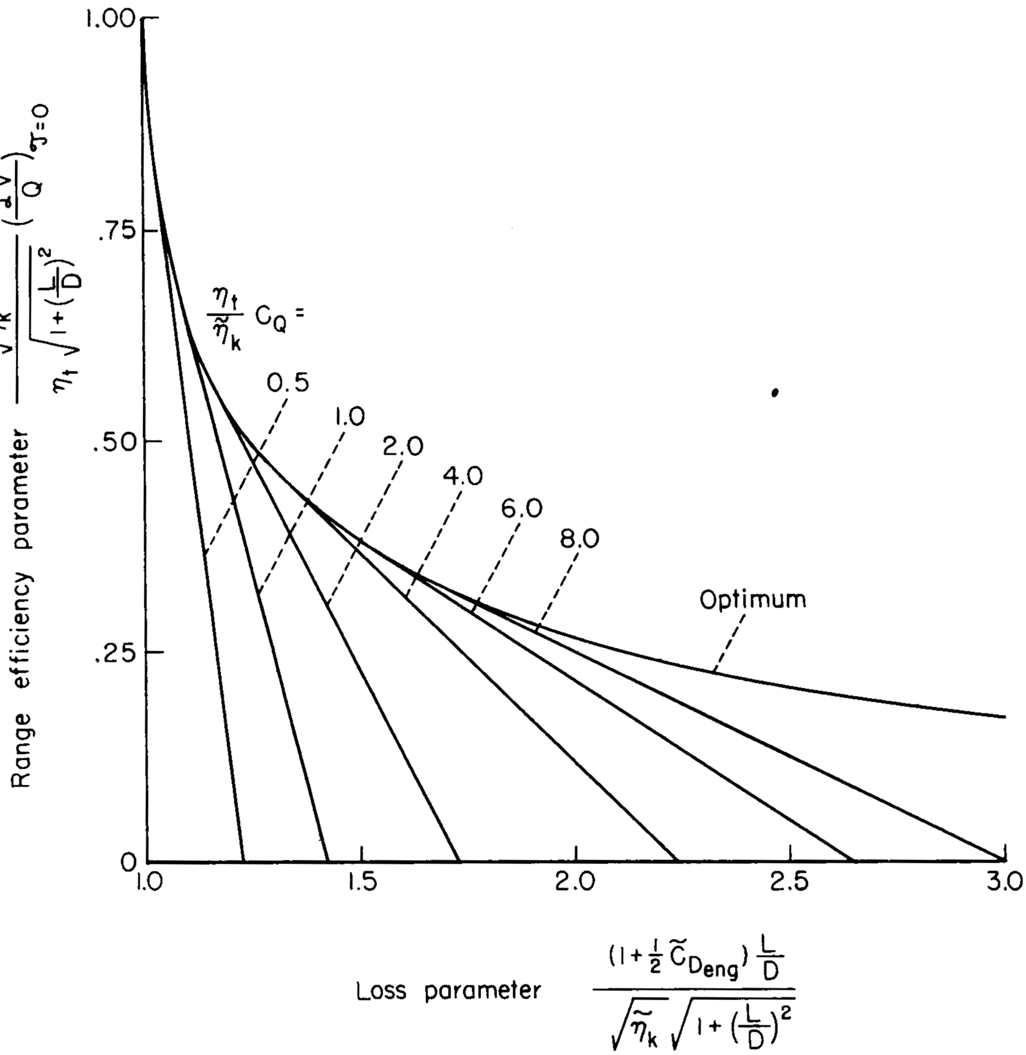

Figure 4.- Range efficiency factor for several values of power coefficient.

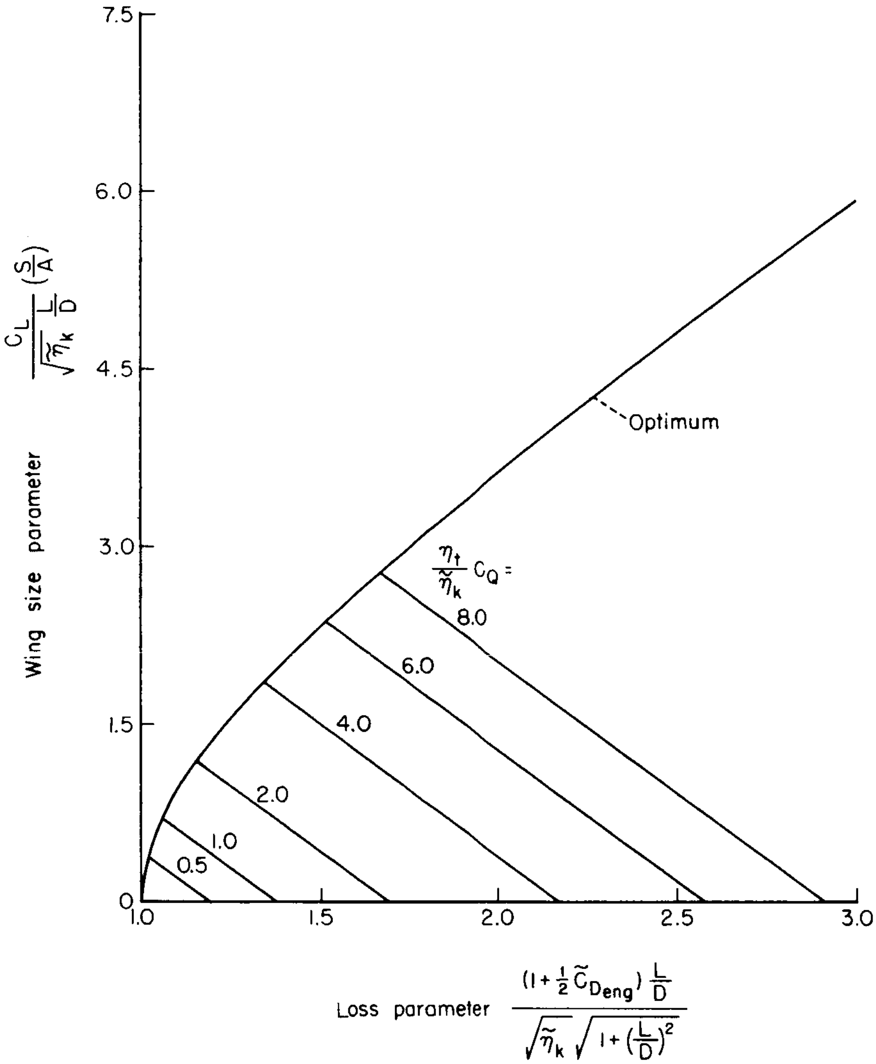

Figure 5.- Optimum ratio of wing plan-form area to engine inlet capture area for several values of power coefficient.

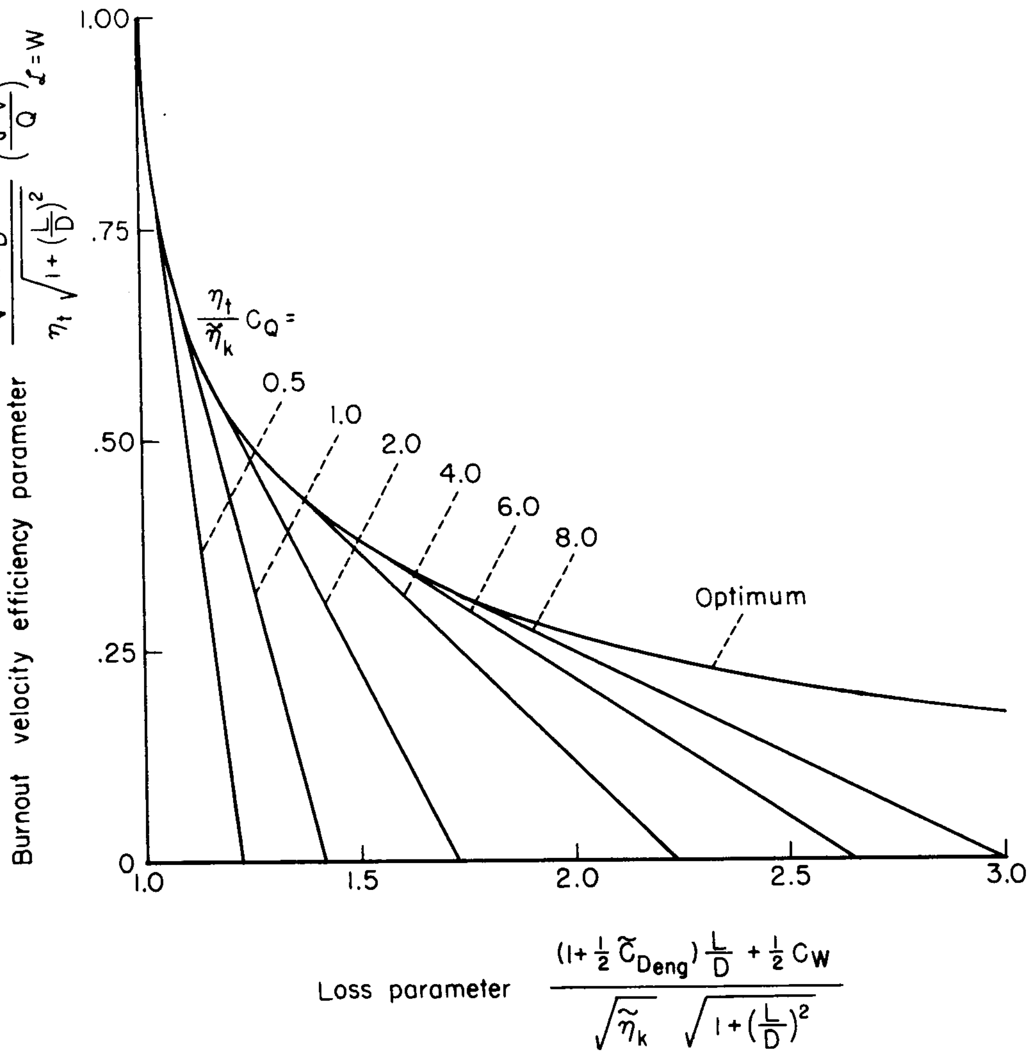

Figure 6.- Over-all airplane efficiency for several values of power coefficient.

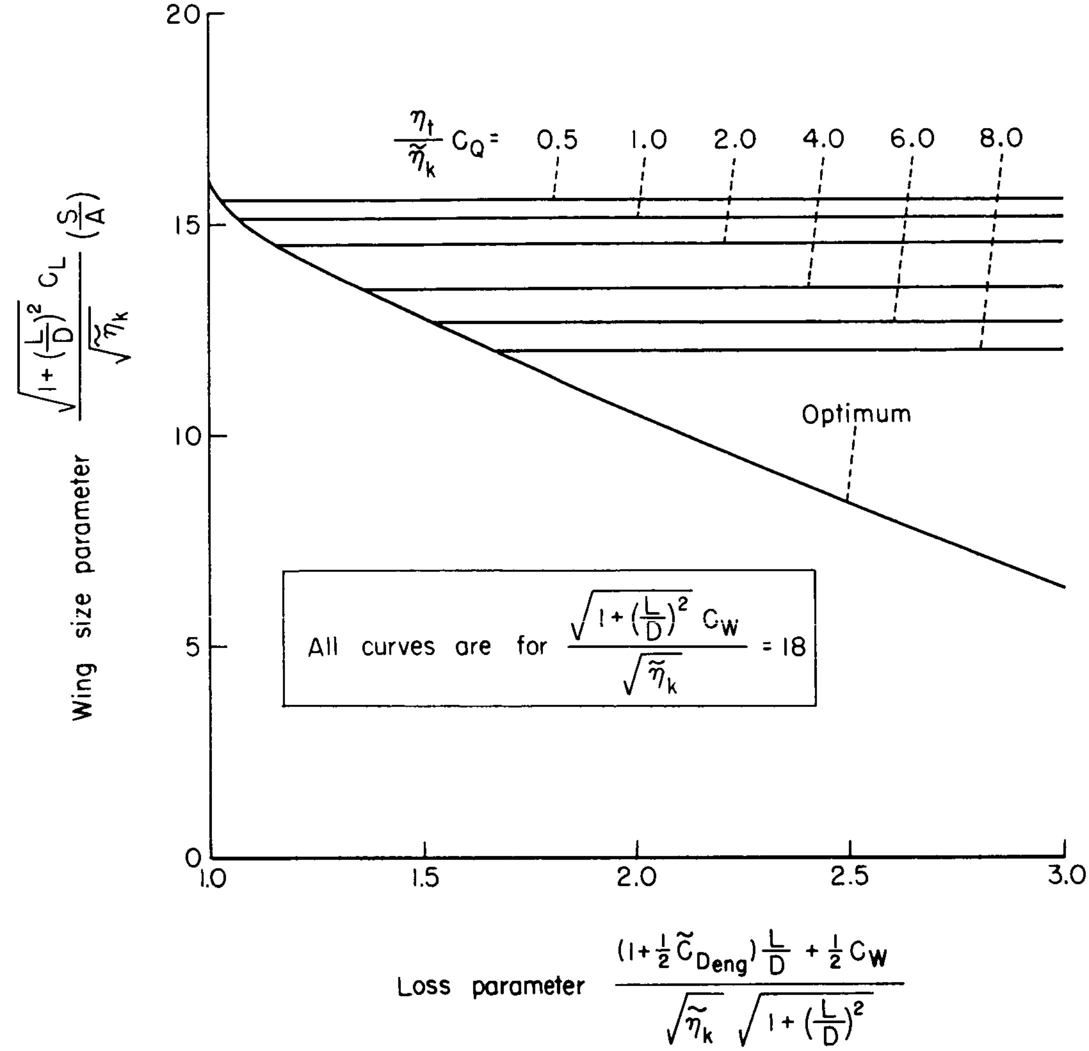

Figure 7.- Ratio of wing plan-form area to engine inlet capture area for maximum burnout velocity.

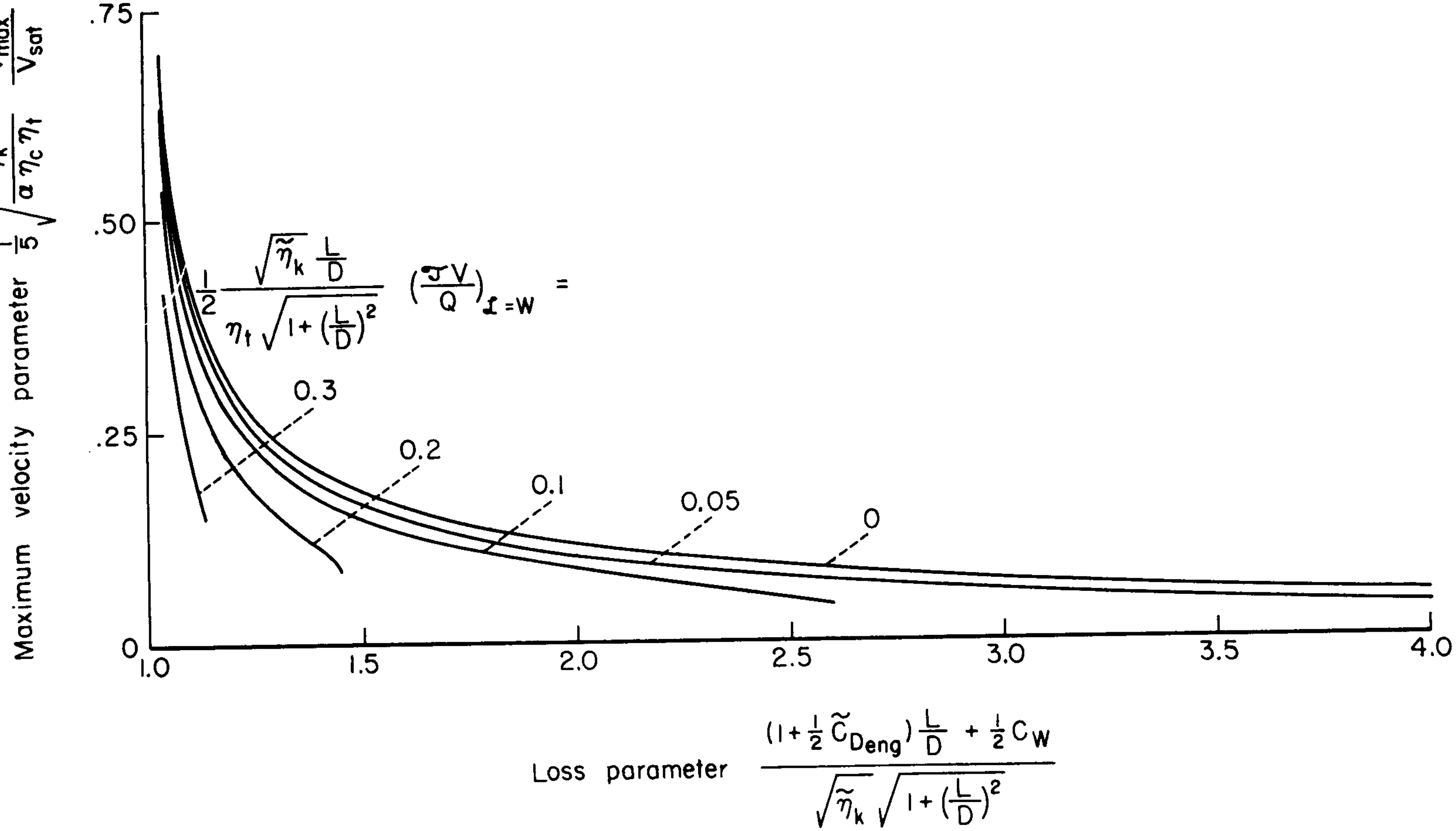

Figure 8.- Maximum vehicle velocity to which a given value of over-all airplane efficiency can be maintained.